Kuwait University **Math 101** **Date:** **January 15, 2011**
Department of Mathematics **Final Exam** **Duration:** **Two Hours**

Calculators, cellular phones and all other mobile communication equipments are not allowed

Answer the following questions. Each question weighs 4 points.

1. Evaluate the following limits, if they exist:

   (a) $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{x^2+x}\right)$.

   (b) $\lim\limits_{x\to 0} x^{\frac{2}{3}}\sin\dfrac{1}{x}$.

2. Suppose that $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ both exist.

   If $\lim\limits_{x\to a}[f(x)+g(x)]=5$ and $\lim\limits_{x\to a}[f(x)-g(x)]=1$, then find $\lim\limits_{x\to a}[f(x)\,g(x)]$.

3. Show that $f(x)=x^5+x^3-1$, has exactly one real root in $[0,1]$.

4. A farmer has 1600 meters of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

5. Find $\dfrac{dy}{dx}$, where

   (a) $y=(x^2-5)^3\sin(x^4-3x+1)$.

   (b) $y=\int_1^{\cos x}\dfrac{1-u^2}{1+u^6}\,du$.

6. Evaluate: $\displaystyle\int\frac{5-4x^3+2x^6}{x^6}\,dx+\int x^{-\frac{3}{4}}\cos\left(2+x^{\frac{1}{4}}\right)\,dx$.

7. Evaluate: $\displaystyle\int_0^2|x^2-1|\,dx$.

8. Find the average value, f_{av}, of $f(x)=\sec^2 x$ over the interval $\left[0,\frac{\pi}{4}\right]$.

9. Find the area of the region bounded by the curves $x=y^2$ and $x=-y^2+2$.

10. The region bounded by the curves $y=4x-x^2$ and $y=3$ is revolved about:

    (a) the line $x=1$,

    (b) the line $y=-1$.

    Set up an integral that can be used to find the volume of the resulting solid in each